AP® Calculus AB
2003 Sample Student Responses
Form B

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A1

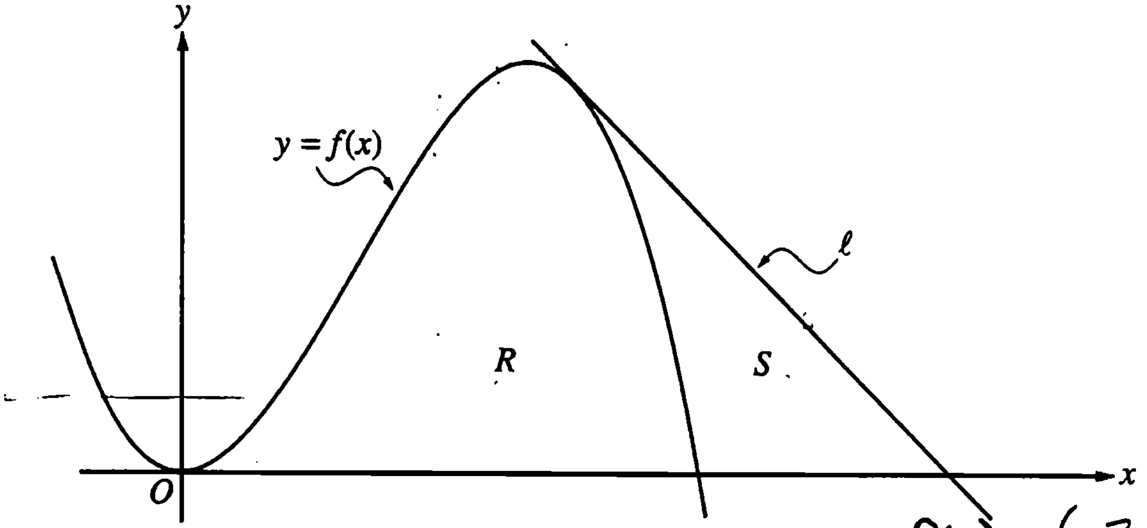
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



points: f(x) = (3, 9)
ℓ = (3, 9)

Work for problem 1(a)

Value for f(x) at x = 3	Value for ℓ at x = 3
$4x^2 - x^3$	18 - 3x
(9) - 27	18 - 9
36 - 27	
= 9	= 9

the points have the same slope and the same point, so they are tangent

slope of f(3)	slope of ℓ = -3
$f'(3) = 8x - 3x^2$	
24 - 27	
= -3	

Continue problem 1 on page 5.

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A2

Work for problem 1(b)

$f(x) = 0 = 4x^2 - x^3$

$4 - x = 0$

$x = 4$

$$\left[\int_3^6 18 - 3x\right] - \left[\int_3^4 4x^2 - x^3\right]$$

$$\left\{18x - \frac{3}{2}x^2\Big]_3^6\right\} - \left\{\frac{4}{3}x^3 - \frac{1}{4}x^4\Big]_3^4\right\}$$

$(54 - 40.5) - (21\tfrac{1}{3} - 15.75)$

$(13.5) - (5.583) = \boxed{7.917 \text{ units}^2}$

Work for problem 1(c)

$(4x^2 - x^3)(4x^2 - x^3)$

$16x^4 - 4x^5 - 4x^5 + x^6$

$$\pi \int_0^4 (4x^2 - x^3)^2$$

$$= \pi \int_0^4 16x^4 - 8x^5 + x^6$$

$$= \pi\left(\frac{16}{5}x^5 - \frac{8}{6}x^6 + \frac{1}{7}x^7\Big]_0^4\right)$$

$$= \pi(156.038)$$

$$= \boxed{490.208 \text{ units}^3}$$

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B,

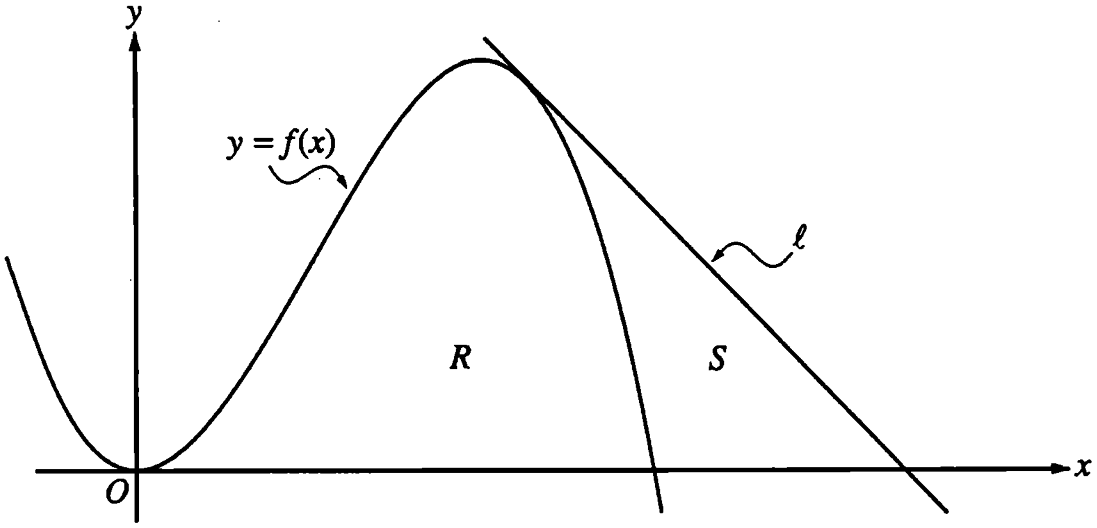
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Both equations have to have same value at 3.

$f(x) = 4x^2 - x^3$ $\quad x=3$

$f(3) = 4(3)^2 - 3^3$

$= 36 - 27$

$= 9$ $\quad$ When $x=3$, $y=9$

$y = 18 - 3x$ $\quad x=3$

$y = 18 - 3(3)$

$y = 18 - 9$

$= 9$ $\quad$ when $x=3$, $y=9$

So ℓ is tangent to the graph of $y=f(x)$ at the point $x=3$.

Continue problem 1 on page 5.

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B_2

Work for problem 1(b)

$f(x) = 4x^2 - x^3$

$0 = 4x^2 - x^3$

$= x^2(4 - x)$

$x = 0, 4$

$y = 18 - 3x$

$0 = 18 - 3x$

$-18 = -3x$

$x = 6$

$S = \int_3^4 [(18 - 3x) - (4x^2 - x^3)]dx + \int_4^6 (18 - 3x)dx$

$= 1.917 + 6$

$= 7.917$

Work for problem 1(c)

$V = \pi \int_0^4 (4x^2 - x^3)^2 dx$

$= \pi \cdot 156.04$

$= 490.21$

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Work for problem 2(a)

$$\int_0^{12} \left(2 + \frac{10}{(1+\ln(t+1))}\right) dt$$

$= 70.571$ gallons

Work for problem 2(b)

$$H(6) = 2 + \frac{10}{(1+\ln(7))}$$

$= 5.395$ gallons coming in

$$R(6) = 12\sin\left(\frac{6^2}{47}\right)$$

$= 8.319$ gallons being removed

The level of heating oil is falling at $t=6$ hours b/c more gallons are being removed than pumped into the tank. $8.319 > 5.395$ or $H(6) < R(6)$ thus lowering the level of heating oil in the tank.

Continue problem 2 on page 7.

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Work for problem 2(c)

$\int_0^{12}\left(2+\frac{10}{(1+\ln(t+1))}\right)dt = 70.571$ gallons pumped in

$\int_0^{12}\left(12\sin\left(\frac{t^2}{47}\right)\right)dt = 73.545$ gallons removed

125 gallons originally

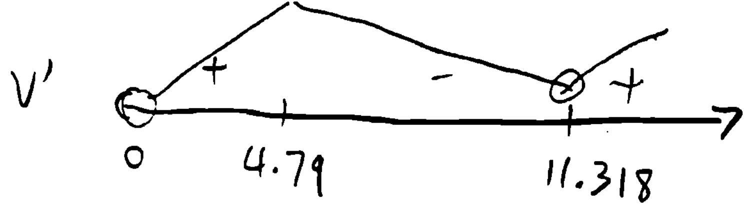
$125 + 70.571 - 73.545 \boxed{= 122.026 \text{ gallons}}$

Work for problem 2(d)

$V = 125 + \int_0^{12}\left(2+\frac{10}{(1+\ln(t+1))}\right)dt - \int_0^{12}\left(12\sin\left(\frac{t^2}{47}\right)\right)dt$

$\frac{dV}{dt} = 2 + \frac{10}{1+\ln(t+1)} - 12\sin\left(\frac{t^2}{47}\right)$

$0 = 2 + \frac{10}{1+(\ln(t+1))} - 12\sin\left(\frac{t^2}{47}\right)$



$V(11.318) = 125+58.118-58.207 = 124.17$

$V(0)=125$

Minimum volume at $t=11.318$. Two local min at $t=0$ and $t=11.318$. $v(0) > V(11.318)$ so $t=11.318$ is when volume of heating oil the least.

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B,

Work for problem 2(a)

$$H(t) = 2 + \frac{10}{1+\ln(t+1)}$$

$$\int_0^{12} 2 + \frac{10}{1+\ln(t+1)}$$

$$= 70.571$$

70.571 gallons of heating oil are pumped in after 12 hours.

Work for problem 2(b)

Adding oil at $2 + \frac{10}{1+\ln(t+1)}$

extracting oil at $12\sin\left(\frac{t^2}{47}\right)$

$$\text{Net Oil } \Delta = \left[2 + \frac{10}{1+\ln(t+1)}\right] - \left[12\sin\left(\frac{t^2}{47}\right)\right]$$

↖ $O(x)$

$$O(6) = 8.594$$

The level of the tank is rising at $t=6$. for we have been given the rates of change for the oil in the tank. As long as $O(t)$ is positive oil is being added to the tank. We have been basically been given the derivative for the amount of oil in the tank

Continue problem 2 on page 7.

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B2

Work for problem 2(c)

At t=12 hours there will be:

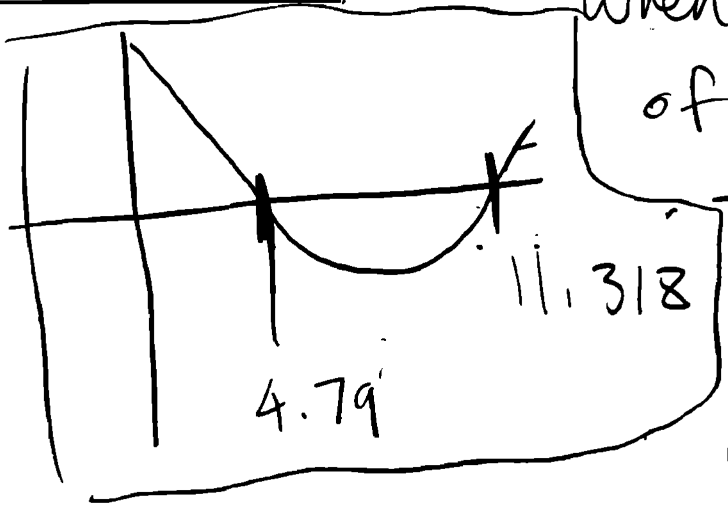
$\int_0^{12} O(t) + 125$ gallons of oil in the tank

$$Vol = \int_0^{12} \left[2+\frac{10}{1+\ln(t+1)} - \left[12\sin\left(\frac{t^2}{47}\right)\right]\right] + 125$$

$$Vol = -2.974 + 125$$

Volume in tank after 12 hours = 122.026 gallons

Work for problem 2(d)



When the derivative (Rate of change of the amount of oil in the tank is at 0, that is when there is a max and a min in the volume in the tank. If there is a sign change from positive to negative it is a max if it is a change from negative to positive it is a min. Therefore O(t) = 0, at two different t values between 0 and 12. 4.79 and 11.318. At 11.318 there is a sign change from negative to positive so that is the location of the minimum point. The volume of oil in the tank at t=11.318 hours is the least amount of oil in the tank

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

Work for problem 3(a)

Since radius $= \frac{1}{2}$(diameter)

$$\Rightarrow \text{Average radius} = \frac{1}{2}\left(\frac{1}{(360-0)\text{mm}}\int_0^{360} B(x)\,dx\right) = \frac{1}{720\text{mm}}\int_0^{360} B(x)\,dx$$

Work for problem 3(b)

$$\int_0^{360} B(x)\,dx = \lim_{\|P\|\to 0}\sum_{k=1}^{3} B(c_k)\,\Delta x$$

$c_1 = 60\text{mm} \Rightarrow B(c_1) = 30\text{mm}$

$c_2 = 180\text{mm} \Rightarrow B(c_2) = 30\text{mm}$

$c_3 = 300\text{mm} \Rightarrow B(c_3) = 24\text{mm}$

$$\Delta x = \frac{360\text{mm}}{n} = \frac{360\text{mm}}{3} = 120\text{mm}$$

$$\Rightarrow \sum_{k=1}^{3} B(c_k)\,\Delta x = 120\text{mm}\left(B(c_1) + B(c_2) + B(c_3)\right)$$

$$= 120\text{mm}\left(30\text{mm} + 30\text{mm} + 24\text{mm}\right) = 10080\text{mm}^2$$

$$\Rightarrow \text{Average radius} = \frac{1}{720}\int_0^{360} B(x)\,dx \approx \frac{1}{720\text{mm}}\left(10080\text{mm}^2\right) = 14\text{mm}$$

Continue problem 3 on page 9.

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A_2

Work for problem 3(c)

It is the volume of blood in the blood vessel starting from a distance of 125 mm from 1 end to a distance of 275 mm from the same end. The units will be $(mm)^3$

Work for problem 3(d)

$b, a, d, c, e, f \in (0, 360)$

$c > f$, $d > e$, $b > a$

$$B''(x) = 0 \Rightarrow \frac{B'(b) - B'(a)}{b - a} = 0$$

$$\Rightarrow B'(b) = B'(a)$$

$$\Rightarrow \frac{B(d) - B(e)}{d - e} = \frac{B(c) - B(f)}{c - f}$$

Since for all x, Δx is the same

$$\Rightarrow B(d) - B(e) = B(c) - B(f)$$

From the table there are values of d, e, c, f such that $B(d) - B(e) = B(c) - B(f)$. For example at $x = 300$

$$B(360) - B(300) = B(300) - B(240) \Rightarrow 26 - 24 = 26 - 24 \Rightarrow 0 = 0$$

END OF PART A OF SECTION II

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B1

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$F_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Work for problem 3(a)

$B(x)_{avg} = \frac{1}{360-0}\int_0^{360} \frac{B(x)}{2}\,dx$

$= \frac{1}{360}\int_0^{360} \frac{Bx}{2}\,dx$

Work for problem 3(b)

$\frac{360}{3} = 120$

$B(x)\ avg = \frac{1}{360}\left[\frac{120 f(60)}{2} + \frac{120\cdot f(180)}{2} + \frac{120\cdot f(300)}{2}\right]$

$= \frac{120}{360}\left[15 + 15 + 12\right]$

$= \frac{\cancel{12}^{1}}{\cancel{36}_{3}} \times 42$

$= 14$ mm

Continue problem 3 on page 9.

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Work for problem 3(c)

$\frac{B(x)}{2}$ = radius of blood vessel

$\pi \int_{125}^{275} \left(\frac{Bx}{2}\right)^2 dx$ Volume of the blood vessel from $x = 125$mm to $x = 275$mm in $(mm)^3$

Work for problem 3(d)

At x where $B''(x) = 0$

There is an inflection on the graph

The sign of $B'(x)$ changes

$B'(x)$ = the change of diameter

From the table we know that when the diameter increases $B'(x) > 0$ when diameter decrease $B'(x) < 0$

$B'(x)$ changes signs

$\therefore B''(x) = 0$

END OF PART A OF SECTION II

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A,

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$v(t) = -1 + e^{(1-t)}$

$a(t) = v'(t) = -1(e^{(1-t)}) = -e^{1-t}$

$a(3) = -e^{1-3} = \boxed{-e^{-2}} = \frac{-1}{e^2}$

Work for problem 4(b)

$v(t)$ + + + + : − − − −

1

$a(t)$ − − − − −

Speed increases when signs are same

$v(t)$

$-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$1 - t = 0$

$t = 1$

$a(t)$

$-e^{(1-t)} = 0$

$e^{1-t} \neq 0$

speed is increasing on $(1, \infty)$

$t = 3$ is within the interval and thus speed is increasing

Continue problem 4 on page 11.

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A2

NO CALCULATOR ALLOWED

Work for problem 4(c)

$v(t) = 0 = -1 + e^{1-t}$

$e^{1-t} = 1$

$1 - t = 0$

$1 = t$

$+ + + \quad - - -$

$v(t)$ ——|——
1

the particle changes direction at t = 1

Work for problem 4(d)

$d(t) = \int_0^1 -1 + e^{1-t} \, dt + -\int_1^3 -1 + e^{1-t} \, dt =$

$-t\Big]_0^1 + -e^{1-t}\Big]_0^1 + t\Big]_1^3 + e^{1-t}\Big]_1^3 =$

$(-1-0) + (-1+e) + (3-1) + (e^{-2}-1) =$

$-1 - 1 + e + 2 + e^{-2} - 1 = \boxed{e + e^{-2} - 1}$

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C,

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$v(t)$ = velocity

$a(t)$ = accel.

$v(t) = -1 + e^{1-t}$

$= e^{1-t} - 1$

$v'(t) = a(t) = e^{1-t}(-1)$

$= -e^{1-t}$

at $t=3$, then, accel. is $-e^{1-3} = -e^{-2} = -\frac{1}{e^2}$

Work for problem 4(b)

at $t = 3$ acceleration is negative and velocity is $-1 + e^{-2} = \frac{1}{e^2} - 1$ which is clearly negative (since $\frac{1}{e^2} < 1$). thus since acceleration is negative velocity is decreasing (becoming more negative). But speed is |velocity| (abs. value of velocity) and thus since velocity becomes more negative speed increases.

Continue problem 4 on page 11.

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Work for problem 4(c)

the particle is going in one direction when v(t) is positive, and another when v(t) is neg. So it changes direction at the zeroes of v(t). Testing points on either side (t=0 and t=2) leads to

$0 = -1 + e^{1-t}$

$e^{1-t} = 1$

$\ln(e^{1-t}) = \ln 1$

$1 - t = 0$

$t = 1$ is a zero.

$v(0) = -1 + e^{1}$

$= e - 1$

$=$ pos because $e > 1$

$v(2) = -1 + e^{-1}$

$= \frac{1}{e} - 1$

$=$ negative since $\frac{1}{e} < 1$

this it changes direction at $t = 1$

Work for problem 4(d)

Total distance traveled over $0 \le t \le 3$ $= \int_0^3 v(t)\,dt$

$= \int_0^3 e^{1-t}\,dt - \int_0^3 1\,dt$

let $1-t = u$

$du = -1$

$= -\int_0^3 e^{u}\,du - \int_0^3 1\,dt$

$= \left[-e^{1-t}\right]_0^3 - \left[t\right]_0^3$

$(-e^{-2} + e^{1}) - (3 - 0)$

$\boxed{e - \frac{1}{e^2} - 3}$ = total dist traveled

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AP® Calculus AB
2003 Sample Student Responses
Form B

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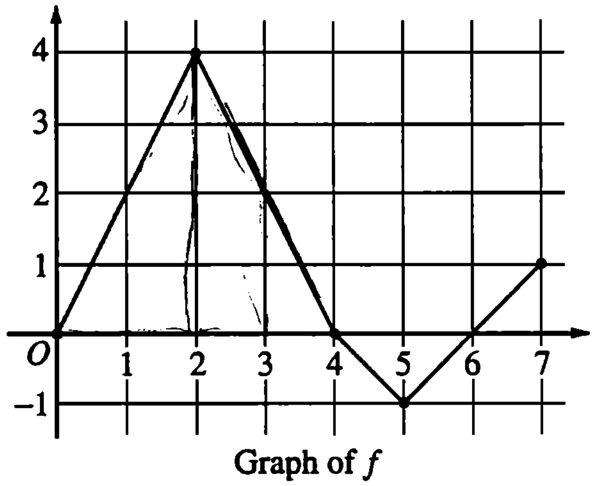

Graph of *f*

Work for problem 5(a)

4+3

$g(x) = \int_2^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g(3) = \int_2^3 f(t)\,dt = F(3) - F(2) = 7 - 4 = 3$

$g'(3) \to f(3) = 2$

$g''(3) = f'(3) = -2$

$g(3) = 3$
$g'(3) = 2$
$g''(3) = -2$

Work for problem 5(b)

rate of change of $g = g'(x)$

$\frac{1}{3-0}\int_0^3 g'(x)\,dx = \frac{1}{3}\{g(3) - g(0)\}$

$= \frac{1}{3}\{3 - g(0)\}$

$= \frac{1}{3}\left(3 - \int_2^0 f(t)\,dt\right)$

$= \frac{1}{3}\left(3 + \int_0^2 F(t)\,dt\right)$

$= \frac{1}{3}\left(3 + F(2) - F(0)\right)$

$= \frac{1}{3}(3 + 4 - 0)$

$= \frac{7}{3}$

$\boxed{\frac{7}{3} \simeq 2.333}$

Continue problem 5 on page 13.

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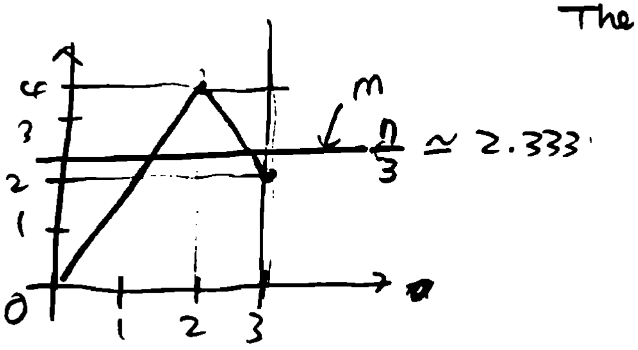
A2

Work for problem 5(c)

$g'(c) = \frac{7}{3}$

since $g'(x)=f(x)$, $g'(c) = f(c)$.

$g'(c) = f(c) = \frac{7}{3}$



The line m crosses the graph of f twice

→ $g'(c)$ is equal to 2.333 at two values of c.

Work for problem 5(d)

At points of inflection, $g''(x)$ should change from (+) to (−), or vice versa.

At $f(2)$, $f'(x)$ changes from (+) to (−), and at $f(5)$, $f'(x)$ changes from (−) to (+).

Points of inflection exist at $x=2$ and $x=5$.

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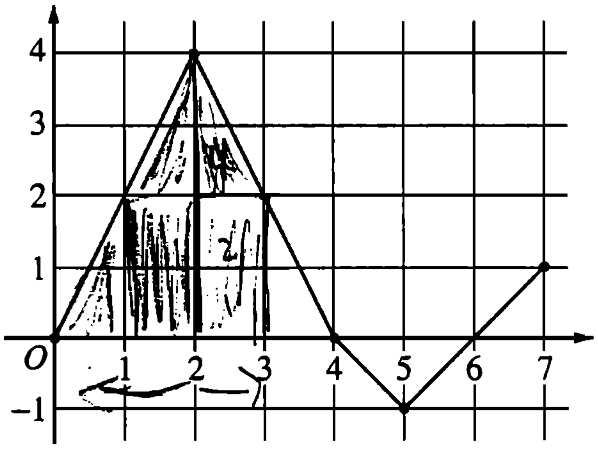
B,



Graph of *f*

Work for problem 5(a)

$$g(3) = \int_2^3 f(t)\,dt = \boxed{3}$$

$'(x)=$ $f(x)$ $\Rightarrow g'(3) = f(3) = \boxed{2}$

$$g''(3) = f'(3) = \text{slope at } 3 = \frac{2-4}{3-2} = \frac{-2}{-1} = \boxed{2}$$

Work for problem 5(b)

avg rate of change $= \dfrac{g(a)-g(b)}{a-b}$

$g(0) = \int_2^0 f(t) = -4$

$g(3) = \int_2^3 f(t) = 3$

$$\frac{g(0)-g(3)}{0-3} = \frac{-4-3}{-3} = \boxed{\frac{7}{3}}$$

Continue problem 5 on page 13.

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B2

Work for problem 5(c)

$g'(c) = 7/3 \Rightarrow$

$f(c) = 7/3$ at 1 (one) point

because

on $(0, 2)$, $f(x) = y = 2x$

$2x = 7/3$

$x = 7/6 \leftarrow$ only at $x = 7/6$

on $(2, 3)$, $f(x) = y = 2x + 8$

$7/3 = 2x + 8$

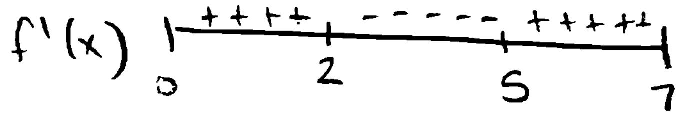
$7/3 - \frac{24}{3} = 2x \Rightarrow -\frac{17}{3} = 2x$ ~~$x = -17/6$~~ (not on $(0, 3)$)

Work for problem 5(d)

point of inflection $= g''(x) = 0$

$g''(x) = f'(x)$

$f'(x) = 0$ at $\boxed{x = 2,\ x = 5}$



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A1

Work for problem 6(a)

$f'(x) = x\sqrt{f(x)}$ $\qquad f(3) = 25$

$= x\left(f(x)^{1/2}\right)$ $\qquad f'(3) = 3\left(f(3)^{1/2}\right) = 3 \cdot 5 = 15$

$$f''(x) = \left(f(x)^{1/2}\right) + x\left(\tfrac{1}{2} f(x)^{-1/2} \cdot f'(x)\right)$$

$$f''(3) = \left(f(3)^{1/2}\right) + 3\left(\tfrac{1}{2} \cdot f(3)^{-1/2} \cdot f'(3)\right)$$

$$= 5 + 3\left(\tfrac{1}{2} \cdot \tfrac{1}{\cancel{5}} \cdot \overset{3}{\cancel{15}}\right)$$

$$= 5 + 3\left(\tfrac{3}{2}\right)$$

$$= 5 + \tfrac{9}{2}$$

$$= \underline{\underline{9\tfrac{1}{2}}}$$

Continue problem 6 on page 15.

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A2

Work for problem 6(b)

$f(3)=25$

$$\frac{dy}{dx} = xy^{1/2}$$

$$\int y^{-1/2}\,dy = \int x\,dx$$

$$2y^{1/2} = \frac{1}{2}x^2 + B \qquad x=3,\ y=25$$

$$2 \cdot 5 = \frac{1}{2}(9) + B$$

$$B = 10 - 4\frac{1}{2} = 5\frac{1}{2} = \frac{11}{2}$$

$$\underline{2y^{1/2} = \frac{1}{2}x^2 + \frac{11}{2}}$$

simplified $y^{1/2} = \frac{1}{4}x^2 + \frac{11}{4}$

$$\therefore\ \underline{\underline{y = \left(\frac{1}{4}x^2 + \frac{11}{4}\right)^2}}$$

END OF EXAMINATION

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C_1

Work for problem 6(a)

$f(3) = 25$

$(3,25)$

$f'(x) = 3\sqrt{25}$

$f'(x) = 3 \cdot 5$

$= 15$

$f''(x) = x \cdot \frac{1}{2}(f(x))^{-1/2} \cdot f'(x) + \sqrt{f(x)} \cdot 1$

$f''(3) = 3 \frac{\cdot 15}{2\sqrt{25}} + \sqrt{25}$

$= 3 \cdot \frac{15}{2 \cdot 5} + 5$

$= \frac{9}{2} + \frac{10}{2} = \boxed{\frac{19}{2}}$

Continue problem 6 on page 15.

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C

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Work for problem 6(b)

$\int \frac{dy}{\sqrt{y}} = \int x\,dx$

$y^{-1/2}$

$2y^{1/2}{}_{+C} = x^2 + C$

$2\sqrt{25} = 3^2 + C$

$2 \cdot 5 = 9 + C$

$10 = \cancel{9} + C$

$-9 \quad \cancel{-9}$

$C = 1$

$25 = 3x \cancel{+1}$

-1

$x = 8$

$\boxed{y = 8x + 1}$

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